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Renormalization-group study of a hybrid driven diffusive system

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We consider a d-dimensional stochastic lattice gas of interacting particles, diffusing under the influence of a short-ranged, attractive Ising Hamiltonian and a "hybrid" external field which is a superposition of a *uniform* and an *annealed random* drive, acting in orthogonal subspaces of dimensions one and m, respectively. Driven into a nonequilibrium steady state, the half-filled system phase segregates via a continuous transition at a field-dependent critical temperature. Using renormalization-group techniques, we show that its critical behavior falls into a new universality class with upper critical dimension $d_c=5-m$, characterized by *two* distinct anisotropy exponents, which, like all other indices, are computed exactly to all orders in perturbation theory.

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I. INTRODUCTION

The study of simple model systems, driven into nonequilibrium steady states by external forces, has received rapidly growing attention over the past decade [1–3]. Characterized by a small number of parameters, these systems allow us to explore their behavior in detail, through analytic and simulational techniques, and thus serve as important tools for deepening our understanding of generic phenomena far from thermal equilibrium. The "standard model" here is the uniformly driven lattice gas which consists of particles hopping to vacant nearest neighbor sites on a fully periodic, d-dimensional square lattice, subject to the usual Ising energetics and a uniform external driving field $E\hat{\mathbf{x}}$, which biases the jump rates along a particular lattice axis $\hat{\mathbf{x}}$ [1]. As in other driven systems, the steady state of the standard model cannot be characterized by a microscopic Hamiltonian *unless* the drive vanishes, in which case it reduces to the equilibrium Ising model [4] with conserved dynamics [5,6]. Focusing on long-wavelength, long-time behavior, considerable progress has been made in understanding the effect of driving forces on correlation functions in the disordered phase [7], interfacial behavior [8], and universal critical properties [9,10]. In particular, the pairing of global anisotropy with a conservation law for the order parameter has been recognized [11] as the crucial factor leading to singular correlations at all temperatures as well as universal behavior *distinct* from the Ising model.

According to the global symmetries of the driving force, different nonequilibrium universality classes emerge. In the case of the standard model, the drive, denoted by $\mathbf{E}$, is *uniform,* resulting in exponents that can be found to all orders in an expansion around the upper critical dimension $d_c=5$ [9]. In contrast, if we consider a lattice gas driven by a *random* field, acting in an m-dimensional subspace with zero average and short-range correlations in space and time, the upper critical dimension changes to $d_c=4-m$, and exponents must be computed order by order in $\epsilon=d_c-d$ [10,12]. We will refer to these two models as the uniformly and randomly driven systems, UDS and RDS, respectively.

In this paper, we focus on the critical behavior of an Ising lattice gas on a three-dimensional cubic lattice driven by an *off-axis* uniform field, $\mathbf{E}=E\cos\theta\hat{\mathbf{x}}+E\sin\theta\hat{\mathbf{y}}$, which biases particle jumps along both the $\hat{\mathbf{x}}$ and the $\hat{\mathbf{y}}$ direction. As has previously been argued [12], such a drive induces a three-fold anisotropy; *effectively,* particles are driven *uniformly* along the $(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}})$ direction, but *randomly* along the $(-\sin\theta\hat{\mathbf{x}}+\cos\theta\hat{\mathbf{y}})$ direction. In the remaining subspace (spanned by $\hat{\mathbf{z}}$), jumps are controlled by energetics only. At half-filling, this system is expected to undergo a continuous phase transition at a critical temperature $T_c(E,\theta)$, from a disordered to a phase-segregated state with interfaces oriented parallel to the x-y plane. In the following, we generalize to a d-dimensional system, driven by a "hybrid" field which consists of two components, one uniform and one random, acting in orthogonal subspaces of dimensions one and m, respectively.

The paper is organized as follows. We first obtain a coarse-grained Langevin equation of motion for our theory. Then, using standard field-theoretic methods [14,15], we derive the critical scaling forms for the vertex functions at the fixed point, focusing specifically on the structure factor and the equation of state. We conclude with a brief summary and some comments.

II. MODEL

In principle, the dynamics of our system can be fully specified in terms of discrete microscopic transition rates and the associated Master equation. In practice, however, such a formulation does not lend itself easily to analytic approaches. A coarse-grained, continuous Langevin equation of motion for the slow variables of the theory is a much more convenient starting point for mean-field and perturbative methods. Given that such a coarse graining can rarely be made explicit, the continuum theory is usually postulated on the basis of symmetry arguments and conservation laws. This approach, which has proven highly successful in the theory of equilibrium critical dynamics, will be followed here also.

For an Ising lattice gas with a fixed number of particles, the only slow variable, after coarse graining, is the order parameter field $\phi(x,t)$, corresponding to the local deviations of the density $\rho(x,t)$ away from its average $\bar{\rho}$. Since we are interested in critical behavior, we choose $\bar{\rho}=\frac{1}{2}$, whence $\phi(x,t)\equiv 2[\rho(x,t)-\frac{1}{2}]$. For other values of $\bar{\rho}$, the system phase segregates via a first order transition. Clearly, $\phi(x,t)$ is conserved, so that the equation of motion takes the form of a continuity equation, $\partial_t\phi+\nabla_j=0$. For vanishing drive, the current is given by Model B of Halperin, Hohenberg, and Ma [6], namely $j=-\lambda\nabla\delta\mathcal{H}/\delta\phi+\zeta$, where $\mathcal{H}$ is the usual Ginzburg-Landau-Wilson Hamiltonian, λ sets the time scale, and $\zeta(x,t)$ denotes a Gaussian distributed white noise modeling the effect of thermal fluctuations. In the presence of a hybrid drive the dynamics is still conserved, but the form of the current changes drastically by virtue of two key effects. (i) The three-fold microscopic anisotropy breaks the invariance of Model B under the full d-dimensional rotation group; thus, all d-dimensional Laplacians split into three components: ∂^2 and ∇_R^2 act in the two (one- and m-dimensional) driven subspaces, while ∇^2 controls the remaining transverse subspace. All carry coupling constants whose ratios may differ from unity. In particular, the extra stirring in the two driven subspaces enhances the corresponding diffusion coefficients, τ_E and τ_R, over the transverse $\tau_\perp$, so that only the latter vanishes at criticality. Consequently, all second order gradient terms (∂^2, ∇_R^2 and ∇^2) appear in the usual Landau expansion, but at fourth order the *transverse* term $(\nabla^2)^2$ suffices in order to stabilize spatially varying density fluctuations near criticality. (ii) The uniform field, acting along unit vector $\hat{\mathbf{e}}$, adds a systematic contribution $j_E=\lambda\sigma(\phi)\mathcal{E}\hat{\mathbf{e}}$ to the current, proportional to $\mathcal{E}$ (which is the coarse-grained E) and a density-dependent conductivity $\lambda\sigma(\phi)$. Due to the excluded volume condition, j_E must vanish if the local density is zero or 1 whence $\sigma(\phi)\propto(1-\phi^2)[1+O(\phi)]$. Summarizing, we obtain the equation of motion,

$$\partial_t\phi(\mathbf{x},t)=\lambda\left[(\tau_\perp-\nabla^2)\nabla^2\phi+\tau_E\partial^2\phi+\tau_R\nabla_R^2\phi+\frac{g}{3!}\nabla^2\phi^3+\mathcal{E}\partial\phi^2\right]-\nabla\zeta\ , \tag{1a}$$

where

$$\langle\nabla\zeta(\mathbf{x},t)\nabla'\zeta(\mathbf{x}',t')\rangle=2\lambda n_\perp(-\nabla^2)\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\ . \tag{1b}$$

If we compare leading terms in the equation of motion near criticality, after a Fourier transform to wave vectors $k\equiv(k_E,k_R,k_\perp)$, we immediately recognize that both k_E and k_R scale as $k_\perp^2$ at the Gaussian level. Thus, operators such as $\partial^2\phi^3$ and $\nabla_R^2\phi^3$, or additional Langevin noise terms $\partial\eta$, $\nabla_R\xi_R$ with correlations n_E, n_R are irrelevant in the renormalization group sense, compared to $\nabla^2\phi^3$ and $n_\perp$. Even though present in principle, they do not contribute to the dominant critical singularities and may therefore be neglected. Simple power counting shows that $\partial\phi^2$ is the leading nonlinearity of the theory, with an upper critical dimension $d_c=5-m$. The second nonlinearity in (1), i.e., $\nabla^2\phi^3$, plays the role of a dangerous irrelevant operator: while naively irrelevant, it must be retained in order to stabilize the ordered phase below criticality. Clearly, we recover the equation for the UDS [9] by setting $m=0$; similarly, the RDS fixed point equation [10] results if we set $\mathcal{E}=0$ and expand the transverse subspace to include $\hat{\mathbf{e}}$.

Expecting mean-field exponents in dimensions $d\geq d_c$ (with logarithmic corrections in $d=d_c$ itself), and noting that our hybrid scenario requires $d\geq m+2$, we recognize that $d=3$, corresponding to $m=1$, is the only "interesting" case. Here, nontrivial exponents emerge in an expansion around the upper critical dimension $d_c=4$. Focusing on this case, we now proceed to compute universal scaling properties and critical exponents, using the standard methods of renormalized field theory [14,15]. Given the similarity of the two theories, the treatment here closely follows that for the UDS [9].

III. RENORMALIZED PERTURBATION THEORY

Introducing the Martin-Siggia-Rose response field $\tilde{\phi}(x,t)$ [16], we rewrite the Langevin equation as a dynamic functional [15],

$$J[\phi,\tilde{\phi}]=\int\int d^dx\,dt\{\tilde{\phi}\partial_t\phi-\lambda\tilde{\phi}[(\tau_\perp-\nabla^2)\nabla^2\phi+\tau_E\partial^2\phi+\tau_R\nabla_R^2\phi+\frac{g}{3!}\nabla^2\phi^3+\mathcal{E}\partial\phi^2]+\lambda n_\perp\tilde{\phi}\nabla^2\tilde{\phi}\}\ , \tag{2}$$

so that all correlation and response functions of the system can be expressed as functional averages with weight $\exp(-J)$.

In addition to dimensional analysis, (2) is invariant under a second scale invariance, corresponding to

$$\begin{aligned}&(k_E,k_R,k_\perp)\to(k_E/\alpha,k_R/\beta,k_\perp)\ ,\quad \phi\to(\alpha\beta^m)^{1/2}\phi\ ,\\ &\tilde{\phi}\to(\alpha\beta^m)^{1/2}\tilde{\phi}\ ,\quad \tau_E\to\alpha^2\tau_E\ ,\quad \tau_R\to\beta^2\tau_R\ ,\\ &\mathcal{E}\to\alpha^{3/2}\beta^{m/2}\mathcal{E}\ ,g\to\alpha\beta^m g\ .\end{aligned} \tag{3}$$

Recognizing that $\mathcal{E}\tilde{\phi}\partial\phi^2$ is a cubic operator so that the expansion will be in powers of $\mathcal{E}^2$, we identify the true dimensionless expansion parameter of the theory as

$$u\equiv G_{md}\mu^{-\epsilon}\tau_E^{-3/2}\tau_R^{-m/2}\mathcal{E}^2\ . \tag{4}$$

Here, G_{md} is a geometric factor depending on both m and d, $\epsilon\equiv 5-m-d$, and μ is an external momentum scale, set by $k_\perp$.

We first focus on scaling in the disordered phase, so that the coupling g may be set to zero. Let $\Gamma_{\tilde{N},N}$ be the one-particle irreducible vertex function with $\tilde{N}$ external $\tilde{\phi}$ legs and N external ϕ legs. In our theory, only $\Gamma_{1,1}$ and $\Gamma_{1,2}$ are primitively divergent. Since each vertex carries a momentum k_E on its $\tilde{\phi}$ leg and $J[\phi,\tilde{\phi}]$ is symmetric under reflections $k_E \to -k_E$, $\mathcal{E} \to -\mathcal{E}$, the divergent parts of $\Gamma_{1,1}$ must be of the form $\Gamma_{1,1}^{\text{div}} \sim k_E^2 u[1+O(u^2)]$. Thus, only τ_E needs to be renormalized in order to render $\Gamma_{1,1}$ finite. Further, J is invariant under the Galilei transformation $\phi(x,t) \to \phi(x+\lambda\mathcal{E}a\hat{e}t,t)+a$, $\tilde{\phi}(x,t)\to\tilde{\phi}(x+\lambda\mathcal{E}a\hat{e}t,t)$ which, in analogy to the uniformly driven case [9], leads to a Ward-Takahashi identity [17] relating $\Gamma_{1,1}$ and $\Gamma_{1,2}$:

$$\Gamma_{1,2}(k,\omega;0,0)=\lambda\mathcal{E}k_E\frac{\partial}{\partial\omega}\Gamma_{1,1}(k,\omega) . \tag{5}$$

As a consequence, *both* $\Gamma_{1,1}$ *and* $\Gamma_{1,2}$ are finite upon renormalization of a single coupling, namely τ_E, which gives rise to the only independent, anomalous exponent appearing in our theory.

Next, we define the renormalized coupling τ_E^R via $\tau_E = Z_\tau \tau_E^R$. Using dimensional regularization and minimal subtraction, a one-loop calculation of $\Gamma_{1,1}$ yields $Z_\tau = 1-(3/4\epsilon)u+O(u^2)$ as well as the geometric factor

$$G_{md}=\frac{\Gamma\left[\frac{3-m-\epsilon}{2}\right]\Gamma\left[1+\frac{\epsilon}{2}\right]}{(4\pi)^{d/2}\Gamma\left[2-m-\frac{\epsilon}{2}\right]} . \tag{6}$$

In the remainder of this paper, all coupling constants and vertex functions will be renormalized ones, so that we may omit the superscript R to economize on notation. The Wilson functions follow as

$$\xi(u) \equiv \mu\partial_\mu(\ln\tau_E)|_{\text{bare}} = -\tfrac{3}{4}u+O(u^2) , \tag{7a}$$

$$\beta(u) \equiv \mu\partial_\mu u|_{\text{bare}} = -[\epsilon+\tfrac{3}{2}\xi(u)]u . \tag{7b}$$

The critical scaling behavior of our theory is controlled by the infrared stable solution of the fixed point equation $\beta(u)=0$, given by $u^* = \frac{8}{9}\epsilon + O(\epsilon^2)$ to one-loop order in dimensions $d<d_c$. We emphasize that the form of $\beta(u)$, Eq. (7b), is completely determined by the symmetries of our model, without any explicit perturbative calculations; thus, it is exact to all orders in u, within perturbation theory. Consequently, we obtain $\xi^* \equiv \xi(u^*) = -\frac{2}{3}\epsilon$ *exactly,* at the infrared stable fixed point $u^* \neq 0$ (note that the Gaussian fixed point $u^*=0$ is always infrared unstable), without having to compute u^* explicitly. Still, the one-loop calculation leading to $Z_\tau = 1-(3/4\epsilon)u$ is not redundant. While the *magnitude* of the $O(u)$ term is indeed unimportant (any numerical prefactors can obviously be absorbed in the definition of G_{md}), its *sign* is crucial: only a *negative* one-loop contribution produces an infrared *stable,* nonvanishing u^*.

The independence of the bare vertex functions on the parameter μ leads to the renormalization group equation for their renormalized counterparts,

$$[\mu\partial_\mu+\beta(u)\partial_u+\tau_E\xi(u)\partial_{\tau_E}] \times\Gamma_{\tilde{N},N}(\{k,\omega\};\tau_\perp,\tau_E,\tau_R,u,\lambda)=0 . \tag{8}$$

In the scaling limit $\mu \ll 1$, the coupling u flows to the infrared stable fixed point u^*, where we can integrate (7a), resulting in $\tau_E(\mu) \propto \mu^{\xi^*}$. Combining the solution to (8) with dimensional analysis and the scale invariance (3), we obtain the full asymptotic scaling behavior of the vertex functions, at the fixed point

$$\Gamma_{\tilde{N},N}(\{k,\omega\};\tau_\perp) = \lambda\mu^p\Gamma_{\tilde{N},N}\left[\left\{\frac{k_\perp}{\mu},\frac{k_E}{\mu^{2-\epsilon^*/2}},\frac{k_R}{\mu^2},\frac{\omega}{\mu^4}\right\};\frac{\tau_\perp}{\mu^{1/\nu}}\right] , \tag{9}$$

where $p=d+m+5-\frac{1}{2}\xi^*-\tilde{N}(d+m+3-\frac{1}{2}\xi^*)+N(d+m-1-\frac{1}{2}\xi^*)$, and we have suppressed all arguments of $\Gamma_{\tilde{N},N}$ that are not affected by rescalings. The most striking feature here is the unconventional, strongly anisotropic scaling of wave vector components in the three subspaces. In order to identify the usual, and any new, exponents for our theory, it is more convenient to specialize to the structure factor, $S(k,t) = \int d\omega\, e^{i\omega t}\Gamma_{2,0}(k,\omega)/|\Gamma_{1,1}(k,\omega)|^2$. In the case of threefold anisotropy, its most general scaling form (see, e.g., [2]) is expected to be

$$S(k_\perp,k_E,k_R,t,\tau_\perp) = \mu^{-2+\eta}S\left[\frac{k_\perp}{\mu},\frac{k_E}{\mu^{1+\Delta_E}},\frac{k_R}{\mu^{1+\Delta_R}},t\mu^z,\frac{\tau_\perp}{\mu^{1/\nu}}\right] , \tag{10}$$

which serves as a *defining relation* for critical indices, including *two* new anisotropy exponents, Δ_E and Δ_R. Exploiting (9), we find specifically for our hybrid model,

$$S(k_\perp,k_E,k_R,t,\tau_\perp)=\mu^{-2}S\left[\frac{k_\perp}{\mu},\frac{k_E}{\mu^{2-\xi^*/2}},\frac{k_R}{\mu^2},t\mu^4,\frac{\tau_\perp}{\mu^2}\right], \tag{11}$$

whence $\Delta_E = 1-\frac{1}{2}\xi(u^*) = 1+\epsilon/3$, $\Delta_R=1$, $\nu=\frac{1}{2}$, $\eta=0$, and $z=4$. Since τ_E is the only coupling that suffers renormalization, Δ_E is the only exponent in our model which develops an anomalous dimension, known *exactly* to all orders in ϵ. All other indices retain their Gaussian values, including $\Delta_R=1$, by virtue of the Gaussian scaling $k_R \propto k_\perp^2$. We should stress, however, that a whole zoo of additional critical exponents can be defined, based on Eqs. (10) and (11); *three* correlation length exponents follow from $k_\perp \propto \tau_\perp^{\nu_\perp}$, $k_E \propto \tau_\perp^{\nu_E}$, and $k_R \propto \tau_\perp^{\nu_R}$, resulting in

$$\nu_\perp = \nu = \frac{1}{2} ,$$

$$\nu_E = \nu(1+\Delta_E) = \frac{1}{2}\left[2+\frac{\epsilon}{3}\right] , \tag{12a}$$

$$\nu_R = \nu(1+\Delta_R) = 1 .$$

Similarly, we obtain *three* dynamical exponents,

$$z_\perp = z = 4\ ,\quad z_E = \frac{z}{1+\Delta_E} = \frac{4}{2+\frac{\epsilon}{3}}\ ,\quad z_R = \frac{z}{1+\Delta_R} = 2\ . \tag{12b}$$

Finally, *six* different η-like exponents characterize the anisotropic power-law decays of the structure factor (unprimed indices) and its Fourier transform, the two-point correlations (primed indices), depending on the subspace (labeled $\perp$, E, and R) along which the power law is measured. They follow from (10) via rescalings:

$$\begin{aligned} &\eta_\perp = \eta\ ,\quad \eta_E = \frac{\eta + 2\Delta_E}{1+\Delta_E}\ ,\quad \eta_R = \frac{\eta+2\Delta_R}{1+\Delta_R}\ ,\\ &\eta'_\perp = \eta + \Delta_E + m\Delta_R\ ,\\ &\eta'_E = \frac{\eta+\Delta_E+m\Delta_R-(d-2)\Delta_E}{1+\Delta_E}\ ,\\ &\eta'_R = \frac{\eta+\Delta_E+m\Delta_R-(d-2)\Delta_R}{1+\Delta_R}\ . \end{aligned} \tag{12c}$$

Once expressed in terms of $\Delta_E = 1-\frac{1}{2}\zeta(u^*) = 1+\epsilon/3$, $\Delta_R = 1$, and $\eta = 0$, all of these are exact to all orders in ϵ.

One further exponent, namely the order parameter index β, remains to be determined. Since our scaling analysis, up to this point, presumed $g=0$, it cannot be used directly to extract information about the ordered phase. Rather, g must be included in the renormalization procedure as a dangerous irrelevant coupling. The exponent β then follows from the renormalized equation of state.

The treatment of the dangerous irrelevant coupling g follows standard techniques [18], and is completely analogous to the uniformly driven case [9]. Since, in principle, all operators of the same naive dimension as $\nabla^2\phi^3$ mix with each other, the first step consists in diagonalizing the *matrix* renormalization conditions, resulting in a set of "eigenoperators" which are invariant under the renormalization group. The associated eigenvalues, evaluated at the fixed point u^*, correspond to the scaling powers of the eigenoperators. Fortunately, the coupling g appears only in a single eigenoperator, whose scaling power $\kappa^* \equiv \kappa(u^*) = 2-\epsilon-\frac{1}{2}\zeta(u^*) = 2(d+m-2)/3$ can be found *exactly*, since it is again completely determined by the combination of Galilei and scaling invariances of the model. Thus the coupling g remains irrelevant at the *nontrivial* fixed point, for all $d > 2-m$.

Next, we introduce a "magnetic" field h into the dynamics, in order to obtain the equation of state [19]. Respecting the conservation law and taking into account that the system orders into "strips" oriented transverse to the nondriven directions, the conjugate field enters the dynamic functional (2) through an additional term $\int d^dx \int dt\, \lambda\tilde{\phi}(x,t)\nabla^2 h(x_\perp)$. The equation of state then follows from the vertex generating functional $\Gamma\{\tilde{\phi},\phi\}$, which may be expanded in a Taylor series around the (nonzero) magnetization M:

$$\begin{aligned} \lambda h &= \frac{\partial}{\partial k_\perp^2}\frac{\delta}{\delta\tilde{\phi}}\Gamma\{\tilde{\phi},\phi\}\bigg|_{k=0,\ \tilde{\phi}=0,\ \phi=M}\\ &= \frac{\partial}{\partial k_\perp^2}\sum_N \frac{M^N}{N!}\Gamma_{1N}(\{k_\perp, k_E=0\ ,\ k_R=0\ ,\ \omega=0\},\tau_\perp,g)\bigg|_{k_\perp=0}\ . \end{aligned} \tag{13}$$

As long as g differs from zero, we can compute the vertex functions in the disordered phase and then analytically continue to $\tau_\perp < 0$. The key point to recognize in (13) is that, for $k_E = k_R = 0$, Γ_{11} and Γ_{13} are the *only* nonvanishing vertex functions appearing on the right hand side, contributing only at the tree level. Thus, (13) reduces to the Landau equation of state, and we find $\beta = \frac{1}{2}$, exact to all orders in ϵ.

For completeness, we also quote the full scaling form of the equation of state,

$$h(M,\tau_\perp,g) = \mu^{(d+m+3-\zeta^*/2)/2} \times h(M/\mu^{(d+m-1-\zeta^*/2)/2}, \tau_\perp/\mu^2, g\mu^{\kappa^*})\ , \tag{14}$$

and remark that, even though β follows from (13) without explicit reference to our expression for κ^*, the latter is required to ensure the consistency of the scaling form (14) with the (exact) Landau form of the scaling *function.*

IV. CONCLUSIONS

To summarize our results, we have analyzed the universal steady-state properties of a hybrid driven diffusive system near its critical point. In dimensions $d < d_c = 5-m$, the scaling behavior of our model is controlled by a nontrivial fixed point, the outstanding feature being the emergence of two anisotropy exponents, $\Delta_E = 1+\epsilon/3$ and $\Delta_R = 1$. These determine the relative scaling of wave vectors k_E and k_R with $k_\perp$, i.e., $k_E \propto (k_\perp)^{\Delta_E}$ and $k_R \propto (k_\perp)^{\Delta_R}$. Only Δ_E develops an anomalous dimension under the renormalization group, known exactly by virtue of an intricate interplay of Galilean and scaling symmetries; the exponents η, ν, z, and β, defined through structure factor and equation of state, retain their mean-field values. Various other indices follow from scaling laws.

Comparing our Δ_E here with its equivalent in the uniformly driven case, we note that both are identical if expressed in terms of the appropriate $\epsilon = d_c - d$. However, the scaling properties of our hybrid model are not just the result of a dimensional shift, since the presence of the randomly driven subspace gives rise to new exponents Δ_R, ν_R, z_R, η_R, and η'_R. Moreover, none of these agree with their counterparts in the pure random case, where both ν and Δ_R develop independent anomalous dimensions.

Setting $\epsilon=1$, we expect our results to capture the universal critical behavior of a lattice gas in $d=3$, driven by an off-axis field in the x-y plane. We predict, e.g., $\Delta_E=\frac{4}{3}$ and $\Delta_R=1$ for the anisotropy exponents and $\eta'_E=\frac{3}{7}$ for the critical spatial decay of the two-point correlations along the uniformly driven direction, in contrast to $\Delta_E=\frac{5}{3}$ and $\eta'_E=0$ for the UDS in $d=3$. Even though a full simulational test of our predictions requires a careful, anisotropic finite size analysis [20], the measurement of η'_E might provide some preliminary insight regarding the different universality classes of the UDS and the hybrid model.

We note, in conclusion, that our results are also applicable to a class of two-temperature lattice gases [21]. In the simplest case, particle-hole exchanges in such a system occur at *two* temperatures $T_\parallel$ and $T_\perp < T_\parallel$, where the former controls the jumps within an m-dimensional, "parallel" subspace, while the latter controls the remaining $(d-m)$-dimensional, "transverse" space. Since such a dynamics is anisotropic and enhances the parallel over the transverse diffusion coefficient, it is expected to belong into the universality class of the RDS [10]. This conjecture is indeed borne out by a sophisticated finite size analysis of high-precision Monte Carlo data [22]. Thus, we expect to be able to observe the universal critical behavior of the hybrid model by simulating a two-temperature model in which one of the transverse directions is driven by a uniform field.

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